

10. The lengths of three ribbons are in the ratio of 4 : 3 : 5. If the length of the shortest ribbon is 12 cm, find the length of the longest ribbon.

*11. A sum of money is divided among Eric, Francis and Gerald in the ratio of 2 : 3 : 5. If Gerald receives \$1800 more than Eric,
(a) how much does Eric receive? (b) what is the total sum of money?

*12. The perimeter of a rectangle is 28 cm. If the ratio of the length to the breadth is 4 : 3, find the dimensions of the rectangle.

*13. The sum of all three angles of a triangle ABC is 180° . If the ratio of the size of $\angle A$ to $\angle B$ to $\angle C$ is 1 : 2 : 3, find the value of $\angle B$.

*14. Christine, Dawn and Ellyn had some funfair tickets costing \$2 each.
Christine sold half of the tickets while Dawn and Ellyn sold the remaining tickets in the ratio of 1 : 3.
If Christine sold 36 tickets more than Dawn, how much money did they collect altogether?

PROJECT

Mathematicians, scientists and artists have been aware of a ratio, known as the **golden ratio**, which is very pleasing to the eyes for many years. The ratio has often appeared in art, architecture and even music.

- Make a summary of the history of the golden ratio by searching through the Internet and library.
- Find out what this ratio is and state a few examples where the golden ratio appears.
- A **golden rectangle** is a rectangle whose ratio of length to width is a golden ratio. Many things around us are golden rectangles. Make a list of common items in our surrounding such as windows, wall paintings, tables, envelopes, playing cards, foolscap papers. Find out which of these items are golden rectangles. Summarise and present your findings in class.
- It is claimed that given a golden rectangle of any size, if the largest square is removed from it, whatever remains is a smaller golden rectangle.

Take a few golden rectangles and verify whether the above claim is correct. Can a given golden rectangle generates an infinite number of smaller golden rectangles? (You may start with a rectangle of size 291 mm by 180 mm. Keep cutting the squares off and see if you keep getting smaller and smaller golden rectangles.)

Percentage

ACTIVITY 1

The chart below shows all the whole numbers from 1 to 100.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Example

Use the chart to help you find the percentage of the numbers that are multiples of 5.

Steps

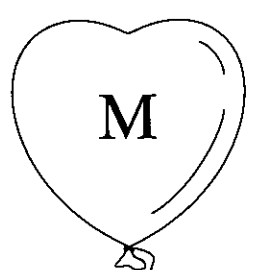
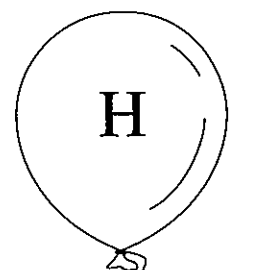
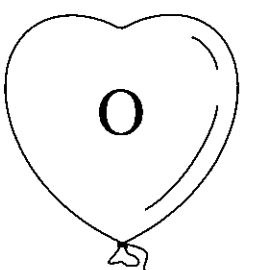
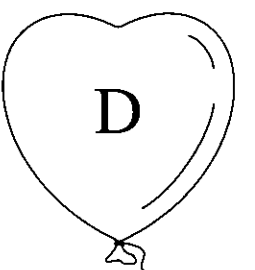
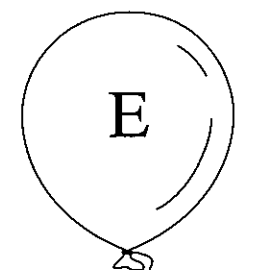
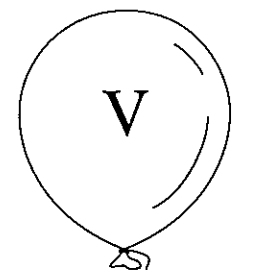
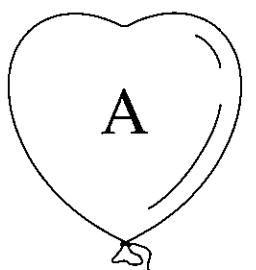
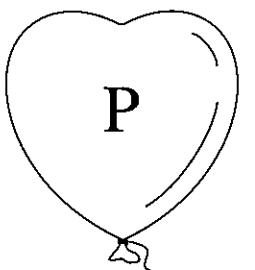
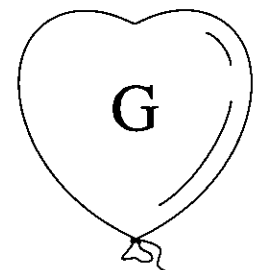
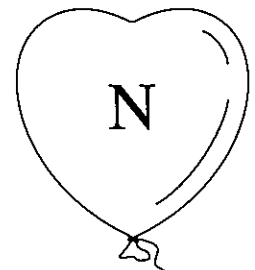
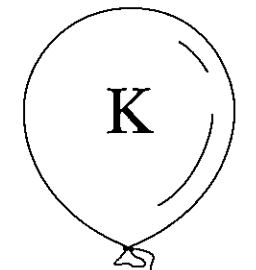
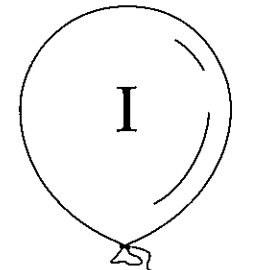
Highlight the multiples of 5 on the chart. You will find that there are 20 out of 100 numbers that are multiples of 5. Therefore, the percentage is 20%.

Use the chart to help you find the percentage of numbers that

- (a) are even numbers, _____
- (b) are multiples of 11, _____
- (c) contain the digit 5, _____
- (d) are factors of 100, _____
- (e) are divisible by 3, _____

ACTIVITY 2

- For each fraction or decimal below, find a balloon with the same value.
- Using the letter on the balloon, write down the hidden message.

 25%	 7%	 50%	 75%
 5%	 40%	 55%	 1%
 100%	 45%	 70%	 10%

- | | | | | | |
|-----------------|------|-----------------|----------------|---------------|---------------|
| $\frac{1}{4}$ | 0.5 | $\frac{2}{5}$ | $\frac{1}{10}$ | 0.45 | $\frac{5}{5}$ |
| — | — | — | — | — | — |
| $\frac{11}{20}$ | 0.07 | $\frac{5}{100}$ | 0.55 | $\frac{3}{4}$ | |
| — | — | — | — | — | |

ACTIVITY 3

- Percentages can be seen in a variety of situations in our daily lives.
- In pairs, cut out 6 to 8 articles that illustrate the different uses of the word 'percentage' or the symbol '%' from newspapers or magazines.
 - Give a short account of how each one was used. (For example, '1.75% – interest rate on a saving account')
 - Make a summary of what you have found and share with your class.

ACTIVITY 4

Mr. Lim and his family went shopping for a sofa set, a bedroom suite and a dining room set. In order to find the best bargain, they visited four different showrooms – Attractive Home, Best Comfort, Comfy Home and Dream Décor.

The prices of the three items at each of the four showrooms are listed below. The prices are not inclusive of 7% GST.

	Sofa set	Bedroom suite	Dining room set
Attractive Home	\$888	\$1515	\$2020
Best Comfort	\$819	\$1180	\$2000
Comfy Home	\$998	\$1395	\$2138
Dream Décor	\$890	\$1109	\$2089

The promotional offers held at each showroom are as follows.

Attractive Home – 20% discount on all prices.

Best Comfort – No GST charges for all items and \$300 cash discount for purchases of 3 items and above.

Comfy Home – 15% discount on all items and no GST charges.

Dream Décor – 20% discount on any item that costs \$1000 and above.

1. From which showroom should Mr. Lim buy
 (a) the sofa set? (b) the bedroom suite? (c) the dining room set?

Explain your answers.

2. If Mr Lim wishes to buy all the three items from one showroom, which one should he buy from, so that he can pay with the least amount of money?

EXERCISE 1

Expressing Percentages as Fractions or Decimals

1. Work out the following.
 (a) Convert 28% to a fraction in its lowest terms.

- (b) Convert 68% to a fraction in its lowest terms.

- (c) Convert $3\frac{1}{5}\%$ to a decimal.

- (d) Convert 2.75% to a decimal.

2. Choose the best answer for each of the following.

- (a) 40% is the same as 10 out of _____.
 A. 10 B. 15 C. 20 D. 25 ()
- (b) 7 out of 25 is equivalent to _____ %.
 A. 7 B. 14 C. 28 D. 56 ()
- (c) Which of the following has a different value from the other three?
 A. 0.72 B. $\frac{18}{25}$ C. $\frac{18}{50}$ D. 72% ()

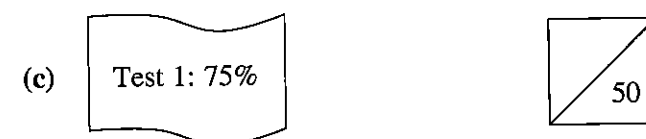
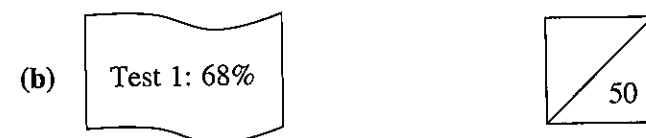
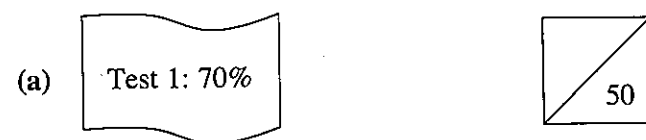
(d) Which of the following is the smallest?

- A. 0.075 B. 7.2% C. $7\frac{1}{4}$ D. $\frac{38}{5}$ ()

(e) Which of the following is not equal to 20%?

- A. 0.20 B. $\frac{2}{200}$ C. $\frac{1}{5}$ D. 20 out of 100 ()

3. Bobby's percentages for three Mathematics tests are given below. Find his original test marks out of 50.



EXERCISES

EXERCISE 2

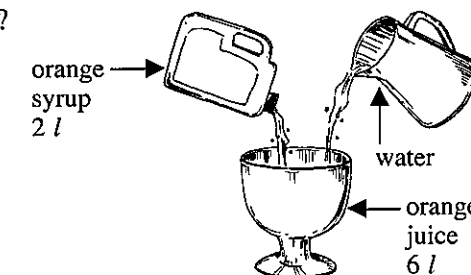
Expressing One Quantity as a Percentage of Another

1. Express as percentage:
(a) 7 of 35
(b) 6 of 300

2. Altogether, 55 out of 216 students in Secondary One failed their NAPFA test (a physical fitness test). What percentage of students passed the test?



3. What is the percentage of water in the orange juice mixture?



4. In a commercial school, there are 90 vacancies for a computer course and 35 vacancies for a designing course. What percentage of the total number of vacancies is allocated for the designing course?

5. 51 out of 68 pupils at a seminar are female. What percentage of the pupils are male?

EXERCISE 3

Dealing with Percentages Greater than 100%

- Change each of the following percentages to a fraction in its lowest terms.

(a) 145%	(b) 425%
(c) 225%	(d) 615%
- Change the following percentages to decimals.

(a) 245%	(b) $400\frac{1}{2}$
(c) 275%	(d) 135%
- Change the following fractions to percentages.

(a) $2\frac{13}{25}$	(b) $1\frac{3}{4}$
(c) 8	(d) $5\frac{7}{20}$
- Change the following decimals to percentages.

(a) 1.15	(b) 2.47
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- As the population of Singapore increases, the demand for housing and other amenities would also increase proportionally. In the year 1985, the total population of Singapore was approximately 2 700 000. In 2005, the total population of Singapore was approximately 4 350 000. Express the total population of Singapore in the year 2005 as a fraction of the total population in the year 1985 and convert the answer to percentage. (Total population comprises all Singapore residents and non-residents.)

EXERCISE 4

Comparing Two Quantities by Percentages

- In Singapore, the average monthly income of a family was \$3076 in 1990. In 2000, the average monthly income of a family rose to \$4940. Express the average monthly income of a family in Singapore in 1990 as a percentage of that in 2000.
- Kwan Meng earns a net amount of \$2500 per month. He gives 5% of his salary to charity every month. How much money does he take home each month?
- This is Ms Chan's telephone bill for January 2005.

Telephone Bill		
Number	Unit	\$
Telephone No. 61234567 – 1 Jan to 31 Jan	3036	21.25
Mobile No. 97654321 Subscription Fee – 1 Jan to 31 Jan		14.00
Off peak	37.2 Min	3.72
Peak	349.6 Min	69.92
Total		

- Calculate the total amount of the bill.
- If Ms Chan had to pay 7% GST on the total amount, calculate the amount of money she had to pay in total.
- If there was a 10% discount off the bill amount due to a promotion, how much would Ms Chan's telephone bill be before including GST?

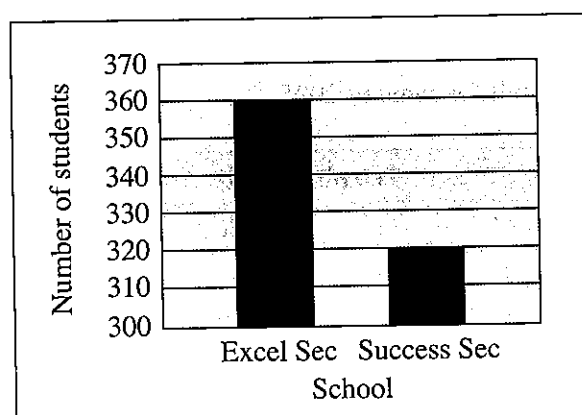
EXERCISE 5

Increasing or Decreasing Percentages

1. Complete the following table.

Original Amount	New amount	Amount increase or decrease	Percentage increase or decrease
\$10	\$11		
50 cm	56 cm		
16 kg	32 kg		
800 g	480 g		
85 ml	68 ml		
60 min	48 min		

2. The enrolments of two private schools, Excel Secondary School and Success Secondary School in January 2004 are shown below.



- (a) Given that the number of students in Excel Secondary School in the following year (January 2005) was 390, find the percentage increase in the number of students.
- (b) If the number of students in Success Secondary School in January 2005 was 120 fewer than that in January 2004, find the percentage decrease in the number of students.

EXERCISE 6

Increasing or Decreasing a Quantity by a Given Percentage

1. During the Great Singapore Sale, Mr Lee bought a handphone at a discount of 20%. The usual price of the handphone is \$288. Calculate the amount paid by Mr Lee.
2. In April 2005, there were 705 000 visitors in Singapore. There was an increase of 16% in the number of visitors in April 2006. Calculate the number of visitors in April 2006.
3. A bag costs 40% less than a pair of shoes. A pair of shoes costs \$45. Calculate
 (a) the cost of the bag,
 (b) the total cost of the two items.

EXERCISE 7

Reversing Percentages

- In an eco-garden, the water level of a lake is measured daily. The water level in the lake on Day 2 increases by 25% to 5 m after a downpour. On Day 3, the water level decreases by 30%.
 - What is the water level on Day 1?
 - What is the water level on Day 3?
- Fiona has 97 stamps. If she gives 80 stamps to Gladys, the number of stamps Gladys has will increase by 320%. How many stamps do they have altogether?
- Mrs Lee wants to buy 4 T-shirts, 3 pairs of shorts, a luggage bag and a pair of slippers. The sale prices of the items before discount are shown in the figure below.

 \$24	 \$9.80
 \$12.50	 \$25



- What is the usual price that she has to pay for all these items?
- Does she qualify for the discount?
- If she qualifies for the discount, how much does she save?

EXERCISE 8

Solving Problems Involving Percentages

- Alan, Brian and Charlie are paid according to the table below. The total sales made were for the month of March.

Salesman	Fixed salary	Commission	Total sales made
Alan	\$620	7%	\$12 000
Brian	\$800	6%	\$16 000
Charlie	\$900	4%	\$20 000

- Calculate the commission made by each of the salesmen.

- Who received the most commission?

- Who had the most earnings in March? How much was his earnings?

- Amanda, Betty and Cathy went on a tour together. During the tour, Amanda spent 20% more than Betty. Betty, however, spent 15% less than Cathy. If Amanda spent \$510 more than Betty, how much did they spend altogether?

REVISION EXERCISE 4

1. Express each of the following as
 - (i) a decimal,
 - (ii) a fraction in its lowest terms.
 - (a) 30%
 - (b) 65%
 - (c) 125%
 - (d) 240%
 - (e) $70\frac{3}{4}\%$
 - (f) $10\frac{1}{5}\%$
 - (g) 45.5%
 - (h) 92.25%
2. There are 468 people living in a block of flats. 25% of the people are men and there are 156 children. Calculate the number of women living in the block.
3. Miss Ng's monthly income is \$1600. She spends 15% of it on food, 20% of it on rent, $7\frac{1}{2}\%$ of it on transport and 40% of the remainder on others. If she saves the rest of her income, find her annual savings.
4. Wati gave 10% of her postcards to Siva, 15% of them to Meena and 10% of them to Zijian. She has 13 postcards left. How many postcards did she give away altogether?
5. May Lien has 10% more stickers than Linda. Rachel has 10% less stickers than Linda. If May Lien has 440 stickers, how many stickers do the three girls have altogether?
6. The United States enforces a Goods-and-Services Tax (GST) of 7%.
 - (a) An item costs US\$380 before GST. What is the cost of the item after GST?
 - (b) Another item costs US\$535 after GST. What is the cost before GST?
7.
 - (a) A television set is put on sale at 28% off its regular price. The sale price is \$379. What was the regular price?
 - (b) Rebecca bought a dress priced at \$125 at a 10% discount. How much did she pay for the dress?

8. During the Great Singapore Sale in July, a departmental store gave a discount of 15% on all items in the store. Mrs. Debbie Yap bought 4 sofa cushions, each priced at \$7, and 2 tablecloths, each priced at \$18. Calculate the amount Mrs. Debbie Yap paid for all these items.
9. Mr Loh, a salesman, has a basic salary of \$950. He is paid a 5.5% commission for every book he sells. If he sells 75 books in the month of November and each book costs \$16, find his income for the month.
10. What is the difference between $\frac{1}{4}$ of 60% of 240 and 35% of 180?
11. Amy's age is 85% of Beatrice's. If their total age is 74, how old is Amy?
12. Mr Goh incurred a loss of 9% when he sold all his books for \$25 935. If he wanted to make a gain of 2%, how much must he sell all his books?
13. Lai Huat is 20% heavier than Maslan. Nigel is 45% heavier than Lai Huat. Express the weight of Nigel as a percentage of the weight of Maslan.
14. A new refrigerator is priced at \$2950. Shop A sells it at 18% discount, while Shop B sells it at a reduction of \$500. What is the difference between the two prices?
15. For every jar of cookies bought, a discount of 10% was given. Mrs Lee bought half a dozen jars of cookies and paid \$7.74 less than usual. How much was each jar of cookies before the discount?

PROJECT

You may have seen some TV guides for different TV channels (e.g. Channel 5, Channel U, Channel News Asia, Central etc.) Each one lists programmes to be aired for every day of a week.

- (a) Gather as many copies of TV guides for different channels as you can for any particular day.
 (b) Construct a table like the one below for each channel, putting each programme into a category (e.g. Sports, News, Education, Kids, Drama etc.).

Name of Channel: Central

Program title	Type	Time

For each of the different tables drawn:

- Calculate the total time allocated to each of the different categories. Hence work out the percentage of time that each category takes up compared to the total broadcasting time.
 - Present your findings in a spreadsheet and share it with your friends. (You may include a column graph of each category against time for display.)
- (c) Discuss any differences found in the percentage for each of the TV channels and suggest a reason for these differences.

Rate and Speed

ACTIVITY 1

Unit Cost Price of Items from Supermarket

You may go to the supermarket to carry out this activity individually or in pairs.

- Find five examples of food items that are packaged in different sizes.
- Calculate the unit cost of each item. The unit cost is found by dividing the cost by the number of units.
- Analyse the data and compare the unit rate to see which item is a better buy.
- Write down your observations. What can you conclude between the size of the item and the unit cost? Is it true that bigger packages are more economical?
- If you are doing this activity with a friend, you may compare the prices of a given item from two supermarkets.

Caution: You may need to convert the items to the same unit before comparing.

ACTIVITY 2

Pulse Rate and Exercise

Objective

To investigate the relationship between pulse rate and exercise

Materials required

Stop watch, graph paper

Procedure

1. Take the pulse of your friend for 20 s. Calculate the pulse rate in beats per minute.
2. Record the pulse rate in a stationary position.

Answers

Revision Exercise 1

- (a) 53, 59, 61, 67
(b) (i) 11 (ii) 97
- (a) 1, 2, 4, 7, 14, 28, 49, 98, 196
(b) $2^6 \times 3^3$
- (a) 4, 720 (b) 24, 720 (c) 6, 360
- (a) 5, 4, 3, 2, -6, -7, -9, -10
(b) 11, 7, -8, -16, -19, -20, -29
(c) 13, 12, 10, 7, -8, -10, -15, -25
- (a) 28 (b) -6 (c) -11
(d) 64 (e) 15 (f) -22
- (a) 112 (b) 231
- (a) -30 (b) -38 (c) 6
(d) -36 (e) 1 (f) 4
- (a) $-1\frac{1}{4}$ (b) $\frac{2}{13}$ (c) -3
- $\frac{2}{13}$, 0.166, 0.17, $\frac{2}{11}$, $\frac{4}{6}$
- (a) 0.582 929, 0.582 999 4, 0.583
(b) 0.03, $\frac{1}{4}$, $\frac{1}{3}$, 0.335, $\frac{5}{11}$
(c) 0.62, 0.623, 0.623 12
- 18°C 12. 334 m 13. -13
- 440 m 15. 53

Revision Exercise 2

- | Number | Nearest 10 | Nearest 100 | Nearest 1000 |
|--------|------------|-------------|--------------|
| 3562 | 3560 | 3600 | 4000 |
| 7008 | 7010 | 7000 | 7000 |
| 9805 | 9810 | 9800 | 10 000 |
| 15 346 | 15 350 | 15 300 | 15 000 |
| 20 532 | 20 530 | 20 500 | 21 000 |
- (a) 5.1 (b) 5.06 (c) 5.063
- | Number | Number of sig. fig. | Number of dec. pl. |
|------------|---------------------|--------------------|
| 74.31 | 4 | 2 |
| 0.023 | 2 | 3 |
| 3.005 7 | 5 | 4 |
| 0.000 467 | 3 | 6 |
| 80 631.025 | 8 | 3 |

- (a) 7300 (b) 17 000
(c) 183.21 (d) 3.735

Number	Correct to			
	2 dec. pl.	2 sig. fig.	Nearest thousand	Nearest whole number
5.049 3	5.05	5.0	5.049	5
16.290 5	16.29	16	16.291	16
0.811 9	0.81	0.81	0.812	1
10.000 5	10.00	10	10.001	10
7.123 4	7.12	7.1	7.123	7

- (a) 78 (b) 28 (c) 1.25
- \$51
- (a) \$1456.30 (b) \$46 000
- (a) \$102 (b) \$110
- \$680
- (a) \$8.70, \$34.80 (b) \$9, \$36
- (a) 40 (b) 50 (c) 6
(d) 0.3 (e) 10 (f) 10
- 25 lumps 14. \$42

Revision Exercise 3

- (a) 1 : 12 (b) 5 : 2
(c) 2 : 9 (d) 2 : 5
- (a) 50 : 13 (b) 13 : 37
- 3 : 6 : 8
- 8
- (a) 98 (b) 168
- $XY = 12$ cm, $ZX = 9$ cm, $YZ = 21$ cm
- 6.4 cm
- (a) \$320 (b) \$720
- (a) 512 (b) 5 : 4
- 20 cm
- (a) \$1200 (b) \$6000
- 8 cm by 6 cm
- 60° 14. \$192

Revision Exercise 4

- (a) (i) 0.3 (ii) $\frac{3}{10}$
(b) (i) 0.65 (ii) $\frac{13}{20}$
(c) (i) 1.25 (ii) $\frac{5}{4}$
(d) (i) 2.4 (ii) $\frac{12}{5}$
(e) (i) 0.707 5 (ii) $\frac{283}{400}$
(f) (i) 0.102 (ii) $\frac{51}{500}$
(g) (i) 0.455 (ii) $\frac{91}{200}$
(h) (i) 0.922 5 (ii) $\frac{369}{400}$
- 195 3. \$6624
- 7 5. 1200
- (a) \$406.60 (b) \$500
- (a) \$526.40 (b) \$112.50
- \$54.40 9. \$1016
- 27 11. 34 years old
- \$29 070 13. 174%
- \$31 15. \$12.90

Revision Exercise 5

- 280 units per month
- \$13.125
- (a) \$45 per hour (b) \$855
(c) 26 hours
- (a) 12 km per litre (b) 96 km
(c) 25 l
- (a) 75 km/h (b) 37.5 km
(c) 2 h 24 min
- (a) 43.8 km (b) 10.95 km/h
- (a) \$18.72 (b) \$56.60
(c) \$90.55
- \$0.95 per kg; \$0.80 per kg; 5-kg pack
- \$1545
- (a) 3 h 45 min (b) 205 km
(c) 54.7 km/h
- \$0.05 per name
- The 4 l for \$3.05 pack

Revision Exercise 6

- (a) $3p + 7$ (b) $18 - 5p$
(c) $4p + \frac{1}{2}q$ (d) $\frac{2p}{9p}$
- 12m cents 3. $\frac{p}{8}$
- (a) 100 (b) 168
(c) 34 (d) 300
- 1st row: 0, 0, 1, 1, 1
2nd row: $-6, -\frac{3}{2}, 5, -8, -2$
3rd row: $20, \frac{5}{4}, 12, -21, 14$
4th row: $8, \frac{1}{2}, -6, 0, 8$
- (a) the number of hours
(b) $C = 6 + 1.5h$
(c) \$52.50
- (a) (i) $P = a + b + 8$ (ii) 24 cm
(b) (i) $P = 2(b + c)$ (ii) 28 cm
(c) (i) $P = 2(a + b)$ (ii) 32 cm
- (a) 57, 105, 193
(b) 5th term
(c) 62, 114, 210
- (d)
- (a) n th term = 4^{n-1}
(b) n th term = n^3
(c) n th term = $44 - 4n$

Revision Exercise 7

- (a) Yes (b) No (c) No
- (a) $\$(9m + 24)$ (b) 6a kg
(c) $6x + 18y$
(d) (i) $\$(4p + 3q)$ (ii) $\$(16p + 12q)$
- (a) $4a + 9$ (b) $8y - z$
(c) $34a$ (d) $\frac{5x}{2}$
- (a) $r + p$ (b) $q - p$
(c) $r + p - q$ (d) $3q - p$
- (a) 10x (b) $9x - 18$
- (a) F (b) F
(c) T (d) F